THE PROPERTY VESTOR

LOYOLA COLLEGE (AUTONOMOUS) CHENNAI - 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER - APRIL 2025



UMT 6501 - COMPLEX ANALYSIS

Date: 23-04-2025 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 PM

	SECTION A - K1 (CO1)
	Answer ALL the Questions - $(10 \times 1 = 10)$
1.	Fill in the Blanks
a)	$Arg\left(\frac{1+i}{1-i}\right)$ is
b)	$\int_{ z =1}^{\infty} \frac{z^2+1}{2z+1} \text{ is } \underline{\hspace{2cm}}.$ If $f(z)=\sum_{n=0}^{\infty} a_n z^n$ then a_n is $\underline{\hspace{2cm}}.$
c)	If $f(z) = \sum_{n=0}^{\infty} a_n z^n$ then a_n is
d)	Residue $\sin\left(\frac{1}{z}\right)$ of at $z=0$ is The function $f(z)=\sin z$ is conformal at
e)	The function $f(z) = \sin z$ is conformal at
2.	True or False
a)	The function $f(z) = \frac{\overline{z}}{z}$ is continuous at $z = 0$.
b)	If u and v are harmonic conjugates to each other in same domain D then u and v must be constant there.
c)	If $u(x,y) = 2(x^2 - y^2) + 3$ for $ z \le 2$ then $\max_{ z =2} u(x,y) = 5$
d)	The transformation $W = \frac{az+b}{cz+d} \ (ad-bc \neq 0)$ where a,b,c,d are complex constants is called linear fractional transformation.
e)	If N is the number of zeros of polynomial $P(z)$ for $ z-a \le R$ then $\int_{ z-a =R} \frac{P'(z)}{P(z)} = N - P.$
	SECTION A - K2 (CO1)
	Answer ALL the Questions $(10 \times 1 = 10)$
3.	Match the following
a)	a) Zero is 1) Harmonic
b)	b) $u(x,y) = 2(x^2 - y^2) + 3$ 2) $2\pi i(N-P)$
c)	c) $\int_{c} \frac{1}{z} dz$; $ z = 1$ 3) Imaginary
d)	d) $w = z + \frac{1}{z}$ has a fixed point at 4) Both real and complex
e)	e) $\int_C \frac{f'(z)}{f(z)} dz$ 5) $2\pi i$

4	Answer the following		
4. a)	C		
(a)	$\left \frac{\operatorname{Re}(z)}{z}\right $; $z \neq 0$		
	Show that $f(z) = \begin{cases} \frac{\operatorname{Re}(z^2)}{ z }; & z \neq 0 \\ 0; & z = 0 \end{cases}$ is cts at $ z = 0$		
	(0; z = 0)		
b)	Find $\int_C \left(\sin(z^2) + \cos(z^2)\right) dz$ where $C: z =1$		
c)	Define residue of a function at a point.		
d)	Determine then singularities of $f(z) = \frac{3z+1}{z^3-i}$ in C .		
e)	Find the fixed point of $w = \frac{(2+i)z-2}{z+i}$.		
	SECTION B - K3 (CO2)		
Ans	wer any TWO of the following $(2 \times 10 = 20)$		
5.	Find all values of $i^{\sin(i)}$.		
6.	Find $\int_{-1}^{1} z^i dz$.		
7.	Obtain the Laurent's series expansion of $f(z) = \frac{-1}{(z-1)(z-2)}$ in the regions		
	(i) $1 < z < 2$ (ii) $ z > 2$.		
8.	Find the Mobius transform which maps 1,-i, 2 on to 0,2,-I respectively.		
	SECTION C – K4 (CO3)		
Ans	wer any TWO of the following $(2 \times 10 = 20)$		
9.	Show that $f(z) = \begin{cases} \frac{\overline{z}}{z}; & z \neq 0 \\ 0; & z = 0 \end{cases}$ satisfies CR equation at (0,0) but not differential at (0,0).		
10.	State and prove Cauchy's residue theorem and hence evaluate $\int_{\mathcal{C}} \frac{5z-2}{z(z-1)} dz$ where C is the circle		
11.	z = 2. Derive Cauchy Riemann equations in polar coordinates for a differentiable function.		
12.	. 1		
12.	Find the value of $\int_{ z =2}^{1} \frac{1}{(z-3)(z^5-1)} dz$		
	SECTION D – K5 (CO4)		
	wer any ONE of the following $(1 \times 20 = 20)$		
13.	State and prove Cauchy integral formula.		
14.	State and prove Taylor's Theorem and obtain the Taylor's series expansion of $\frac{1}{1-z}$.		
	SECTION E – K6 (CO5)		
	wer any ONE of the following $(1 \times 20 = 20)$		
15.	Prove Liouvilles theorem and deduce fundamental theorem of algebra		
16.	Evaluate $\int_{-\infty}^{\infty} \frac{\cos x}{(x^2 + 4^2)(x^2 + 3^2)} dz$.		

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